

MID TERM EXAMINATION
APPLIED MATHEMATICS (241)
SET – II

7/9/2025

Time Allowed: 3 Hours

Maximum Marks:80

General Instructions:

Read the following instructions very carefully and strictly follow them:

- (i) This Question paper contains **38** questions. All questions are **compulsory**.
- (ii) Question paper is divided into **five** Sections – Section **A, B, C, D and E**.
- (iii) In Section **A** – Question Number **1 to 18** are Multiple Choice Questions (MCQs) and Question Number **19 & 20** are Assertion – Reason based questions of **1** mark each.
- (iv) In Section **B** – Question Number **21 to 25** are Very Short Answer (VSA)–type questions, carrying **2** marks each.
- (v) In Section **C** – Question Number **26 to 31** are Short Answer (SA)–type questions, carrying **3** marks each.
- (vi) In Section **D** – Question Number **32 to 35** are Long Answer (LA)–type questions, carrying **5** marks each.
- (vii) In Section **E** – Question Number **36 to 38** are case study based questions, carrying **4** marks each.
- (viii) There is no overall choice. However, an internal choice has been provided in **2** questions in Section – **B**, **2** questions in Section – **C**, **2** questions in Section – **D** and **2** questions in Section – **E**.
- (ix) Use of calculator is **NOT** allowed.

SECTION–A

This section comprises of Multiple Choice Questions (MCQs) of 1 mark each.

Select the correct option (Question 1 to Question 18)

1. If $100 \equiv x \pmod{7}$, then the least positive value of x is
- | | |
|------|------|
| A. 6 | B. 3 |
| C. 4 | D. 2 |

23. From a cylindrical drum containing petrol and kept vertical, the petrol is leaking at the rate of $10 \text{ cm}^3/\text{sec}$. If the radius of the drum is 25 cm and the height is 1 m, find the rate at which the level of the petrol is changing when the petrol level is 80 cm.

24. A. How much money should be invested by a man now to ensure an annual pension of ₹ 24,000 to be received at the beginning of each year forever after his retirement if money is worth 6% compounded annually?

OR

- B. A person invested ₹ 5000 in a fund for 5 years. The value of the investment was ₹ 4800 at the end of the second year, ₹ 6000 at the end of the third year, ₹ 6700 at the end of the fourth year and on maturity, the final value of investment stood at ₹ 8000. Find the CAGR. [Given $(1.6)^{1/5} = 1.908$]

25. Evaluate: $\int \frac{(x^4 - x)^{1/4}}{x^5} dx$

SECTION-C

Question Number 26 to 31 are Short Answer (SA) type questions of 3 marks each.

26. A vessel contains a mixture of two liquids A and B in the ratio 7:9. From this vessel, 16 litres of mixture is drawn off and 16 litres of liquid A is filled in the vessel. If the new ratio of liquids A and B are 11:9, then how many litres of liquids A and B were contained in the mixture initially.

27. If $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$, show that $A^3 - 23A - 40I = 0$.

28. A. Find the points of local maxima and minima (if any) for $f(x) = x^3 - 3x + 3$. Also find the maximum and minimum values.

OR

- B. For manufacturing a certain item, the fixed cost is ₹ 6500 and the cost of producing each unit is ₹ 12.50.

- (i) What is the cost function?
(ii) What is the total cost of producing 75 items?
(iii) What is the average cost of producing 400 items?

29. A. Solve the following linear programming problem graphically:

$$\text{Maximize } Z = 300x + 190y$$

subject to constraints:

$$x + y \leq 24, 2x + y \leq 32, x, y \geq 0$$

OR

B. A manufacturer has three machines I, II, and III installed in his factory. Machines I and II are capable of being operated for almost 12 hours whereas machine III must be operated at least 5 hours a day. He produces only two items M and N, each requiring the use of all three machines. The number of hours required for producing 1 unit of each of M and N on the three machines are given in the following table:

Items	Number of hours required on machines		
	I	II	III
M	1	2	1
N	2	1	1.25

He makes a profit of ₹ 600 and ₹ 400 on each item M and N respectively. How many units of each item should he produce so as to maximize his profit assuming that he can sell all the items that he produces? Formulate a linear programming problem.

30. A man borrows ₹ 10,00,000 and agrees to pay interest quarterly at an annual rate of interest of 8%. At the same time, he sets up a sinking fund in order to repay the loan at the end of 5 years. If the sinking fund earns interest at 6% per annum, compounded quarterly, find the quarterly cost of the debt. {Given $(1.015)^{20} = 1.346852$ }

31. The profit of a soft drink firm (in thousands of rupees) during each month of the year is as given below:

Months	Jan	Feb	March	April	May	June	July	Aug	Sept	Oct	Nov	Dec
Profit	3.6	4.3	4.3	3.4	4.4	5.4	3.4	2.4	3.4	1.8	0.8	1.2

Calculate the four yearly moving averages.

SECTION-D

Question Number 32 to 35 are Long Answer (LA) type questions of 5 marks each.

32.

(A)

An automobile company uses three types of steel S_1 , S_2 and S_3 for producing three types of cars C_1 , C_2 and C_3 . Steel requirements (in tonnes) for each type of cars are given below:

Steel	Cars		
	C_1	C_2	C_3
S_1	2	3	4
S_2	1	1	2
S_3	3	2	1

Using Cramer's rule, find the number of cars of each type which can be produced using 29, 13 and 16 tonnes of steel of three types respectively.

OR

B. If $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$, find A^{-1} and hence solve the following system of

equations:

$$x + 2y + z = 4, \quad -x + y + z = 0, \quad x - 3y + z = 2$$

33.

(A)

Calculate the five yearly moving averages for the following data of the number of commercial and industrial failures in a country from 1992 to 2007:

Year	1992	1993	1994	1995	1996	1997	1998	1999
Number of failures	23	26	28	32	20	12	12	10
Year	2000	2001	2002	2003	2004	2005	2006	2007
Number of failures	9	13	11	14	12	9	3	1

OR

B. Fit a straight line trend by the method of least squares for the following data and find the trend values:

Year	2013	2014	2015	2016	2017	2018	2019
Production (in '000 units)	80	90	92	83	94	99	92

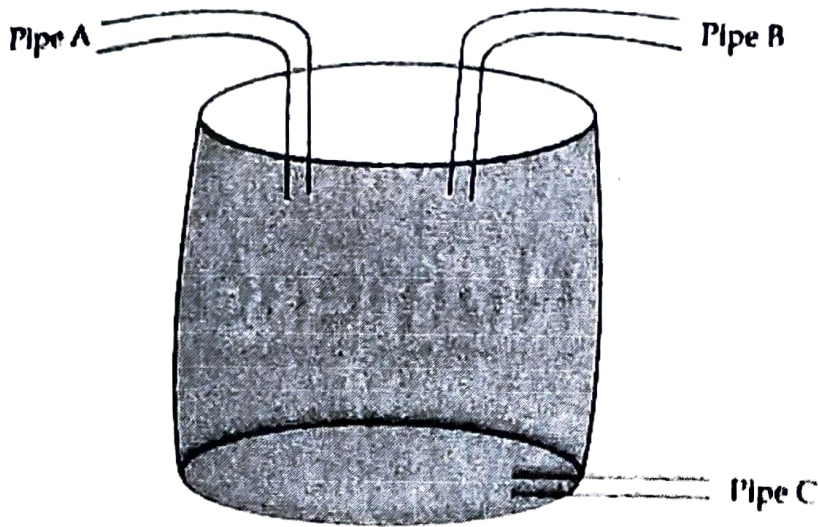
34. A bond of face value ₹ 1000 matures in 5 years. Interest is paid semi-annually and bond is priced to yield 8% p.a. If the present value of bond is ₹ 800, find the annual coupon rate.

35. Find the area enclosed by the parabola $y^2 = x$ and the line $x + y = 2$.

SECTION-E

Question Number 36 to 38 are case-study based questions of 4 marks each.

36. An overhead water tank has three pipes A, B and C attached to it. The inlet pipes A and B can fill the empty tank independently in 15 hours and 12 hours respectively. The outlet pipe C alone can empty a full tank in 20 hours.



Based on the above information, answer the following questions:

- (i) For a routine cleaning of the tank, the tank needs to be emptied. If pipes A and B are closed at the time when the tank is filled to two-fifth of its total capacity, how long will pipe C take to empty the tank completely? (1)
- (ii) How long will it take for the empty tank to fill completely, if all the three pipes are opened simultaneously? (1)
- (iii) ~~A~~ On a given day pipes A, B and C are opened (in order) at 5 a.m., 8 a.m. and 9 a.m. respectively to fill the empty tank. In how many hours will the tank be filled completely? (2)

OR

~~B.~~ Given that the tank is half-full, only pipe C is opened at 6 a.m. to empty the tank. After closing pipe C and an hour's cleaning time, tank

is filled completely by pipes A and B together. What is the total time taken in the whole process? (2)

37. A person amortizes a loan of ₹ 15,00,000 from a bank at an interest of 12% per annum for 8 years, to be paid in monthly instalments.

Based on the above information, answer the following questions:

- (i) Write the formula for calculating EMI by reducing balance method. (1)
(ii) Write the values of P, i and n respectively. (1)
(iii) A. Find the EMI. [Use $(1.01)^{96} = 2.5993$] (2)

OR

- B. If the loan is to be returned in 20 years, find EMI.

[Use $(1.01)^{-240} = 0.09180$] (2)

38. A factory manufactures tennis rackets and cricket bats. A tennis racket takes $1\frac{1}{2}$ hours of machine time and 3 hours of craftsmanship in its making; while a cricket bat takes 3 hours of machine time and 1 hour of craftsmanship. In a day, the factory has availability of not more than 42 hours of machine time and 24 hours of craftsmanship. Profit on a racket and on a bat is ₹ 20 and ₹ 10 respectively.

Based on the above information, answer the following questions:

Let x and y be the number of rackets and bats manufactured by the factory.

- (i) Formulate the above as a linear programming problem to maximise the profit. (2)
(ii) Determine the maximum profit (in ₹) earned by the factory. (2)