BAL BHAVAN INTERNATIONAL SCHOOL

SEC-12 DWARKA, New Delhi-75

Half Yearly Examinations [2025-26]

"Mathematics gives us hope that every problem has a solution"

SET I

Name: _________ Roll No._____
Class: XII Time -3hrs
Subject: Mathematics MM - 80

General Instructions

- **a.** ***This question paper contains five Sections A, B, C, D and E. Each part is compulsory.***
- **b.** ***Section A has 20 very short answer type (SA1) questions of 1 mark each.***
- **c.** ***Section B has 6 short answer type (SA2) questions of 2 marks each.***
- **d.** ***Section C has 5 long answer type (LA) questions of 3 marks each.***
- **e.** ***Section D has 4 long answer type (LA) questions of 5 marks each.***
- **f.** ***Section E has 3 case study based questions of 4 marks each.***

SECTION B

Q.1. If A = $\begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix}$ $then\ A^T + A = I_2\ if$

(a) $\theta = n\pi, n \in Z$ (b) $\theta = (2n+1+)\frac{\pi}{2}, n \in Z$ (c) $\theta = 2n\pi + \frac{\pi}{3}, n \in Z$ (d) none of these

Q.2. If $\begin{bmatrix} 2+x & 3 & 4 \\ 1 & -1 & 2 \\ x & 1 & -5 \end{bmatrix}$ is a singular matrix, then x=

(a) $\frac{13}{25}$ (b) $\frac{-25}{13}$ (c) $\frac{5}{13}$ (d) $\frac{25}{13}$

Q.3. If sin(x+y)=log (x+y), then $\frac{dy}{dx}$ =

(a)2 (b)-2 (c)1 (d)-1

Q.4. If x = $acos^3\theta, y = asin^3\theta\ then\ \sqrt{1+\left(\frac{dy}{dx}\right)^2}$ =

(a) $tan^2\theta$ (b) $sec^2\theta$ (c)sec θ (d)$|sec\theta|$

Q.5. $\frac{d}{dx}\{log_5(logx)\}$ equals

(a) $\frac{1}{log5x}$ (b) $\frac{1}{xlog5logx}$ (c) $\frac{1}{5\log(logx)}$ (d) $\frac{1}{5logx}$

Q.6. If $y = \sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ then $\frac{dy}{dx}$=

(a)$-\frac{2}{1+x^2}$ (b) $\frac{2}{1+x^2}$ (c) $\frac{1}{2-x^2}$ (d) $\frac{2}{2-x^2}$

Q.7. A man 2 metres tall walks away from a lamp post 5 metres height at the rate of 4.8 km/hr. The rate of increase of the length of his shadow is

(a) 1.6 km/hr (b) 6.3 km/hr (c) 5 km/hr (d) 3.2 km/hr

Q.8. IF A and B are square matrices of order 3 such that AB =6I. if , $|A|$=12 then = $|B|$=

(a) $\frac{1}{2}$ (b)2 (c)18 (d)54

Q.9. If $\tan^{-1} x = \frac{\pi}{10}$ $for\ some\ x \in R, then\ the\ value\ of$ $\cot^{-1} x$

(a) $\frac{\pi}{10}$ *(b)* $\frac{2\pi}{5}$ *(c)* $\frac{3\pi}{5}$ *(d)* $\frac{4\pi}{5}$

Q.10. The value of expression $\sin\{\cot^{-1}(\cos(\tan^{-1} 1))\}$ *is*

(a)0 (b)1 (c)$\frac{1}{\sqrt{3}}$ (d)$\sqrt{\frac{2}{3}}$

Q.11. The corner points of the feasible region in graphical representation of a L.P.P. are (2, 72), (15, 20) and (40, If z = 18x + 9y be the objective function, then

(a) Z is maximum at (2, 72), minimum at (15, 20)
(b) Z is maximum at (15, 20), minimum at (40, 15)
(c) Z is maximum at (40, 15), minimum at (15, 20)
(d) Z is maximum at (40, 15), minimum at (2.72)

Q.12. If the feasible region of a linear programming problem with objective function Z = ax + by is bounded, then which of the following is correct?

(a) It will only have a maximum value
(b) It will only have a minimum value.
(c) It will have both maximum and minimum values.
(d) It will have neither maximum nor minimum value.

Q.13. The function $f(x) = -\frac{x}{2} + sinx$ defined on $\left[-\frac{\pi}{3}, \frac{\pi}{3}\right]$

(a)increasing (b)decreasing (c)constant (d)none of these

Q.14. The least value of the function f(x) = x^3 - 18x^2 + 96x in the interval [0,9] is

(a)126 (b)135 (c)160 (d)0

Q.15. f(x)= sinx +$\sqrt{3}cosx$ is maximum when x =

(a) $\frac{\pi}{3}$ *(b)* $\frac{\pi}{4}$ *(c)* $\frac{\pi}{6}$ *(d)0*

Q.16. $\int e^{-logx}\, dx$ equals to

$(a) - e^{-logx}$ +c (b) $-\frac{1}{x}e^{-logx}$ +c *(c)* $logx$ +c *(d)* $-logx$+c

Q.17. The value of $\int cosxe^{sinx}$ *is*

(a) e^{cosx} + c (b) e^{sinx} +c (c) $-e^{sinx}$ +c (d)none of these

Q.18. The value of $\int \frac{1}{x+xlogx} dx$ *is*

(a) 1+logx +c (b) x+logx +c (c) xlog(1+logx)+c (d)log(1+logx)+c

DIRECTION FOR Q19 and Q20. In the question number 19 and 20, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice.
1) Both A and B are true and R is the correct explanation of A.
2) Both A and B are true and R is not the correct explanation of A.
3) A is true and B is false
4) A is false and B is true.

Q.19. Assertion (A): the domain of the function $f(x) = \cos^{-1}(3x+1)\ is\ [-2/3, 0]$
Reason(R): domain of $\cos^{-1} x\ is [-1,1]$

Q.20. Assertion (A): f(x)= tanx –x always increases.
Reason (R): Any function y = f(x) is increasing if $\frac{dy}{dx} \geq 0$

SECTION B

Q21. $If\ cos^{-1}x + cos^{-1}y + cos^{-1}z = \pi, prove\ that\ x^2 + y^2 + z^2 + 2xyz = 1$

Q22. Find the matrix X such that: $\begin{bmatrix} 2 & -1 \\ 0 & 1 \\ -2 & 4 \end{bmatrix} X = \begin{bmatrix} -1 & -8 & -10 \\ 3 & 4 & 0 \\ 10 & 20 & 10 \end{bmatrix}$

Q23. Differentiate $\tan^{-1}(\frac{2^{x+2}}{1-4^x})$, wrt x where $x \neq 0$

Q24. Find $\int \sqrt{cosecx - 1}\, dx$.

Q25. At what point of the ellipse $16x^2 + 9y^2 = 400$ does the ordinate decreases at the same rate at which the abscissa increases.

SECTION C

Q26. If $(\tan^{-1} x)^2 + (\cot^{-1} x)^2 = \frac{5\pi^2}{8}$ then find the value of x

Q27. If $y = (x + \sqrt{x^2+1})^m$, show that $(x^2 + 1)y_2 + x\, y_1 - m^2 y = 0$.

Q28. Find $\int \frac{\cos(x+a)}{\sin(x+b)} dx$

Q29. Find the intervals in which the function given by $(x) = sin4x - cos4x$ is strictly increasing or strictly decreasing when $x \epsilon\ [0, \pi]$.

Q30. Integrate $\int \frac{x^4\, dx}{(x^2+2)(x-3)}$.

Q31. If A = $\begin{bmatrix} 0 & -tan\frac{\alpha}{2} \\ tan\frac{\alpha}{2} & 0 \end{bmatrix}$ and I is the Identity Matrix of of order 2 then,
Prove that : A + I = (I – A) $\begin{bmatrix} cos\alpha & -sin\alpha \\ sin\alpha & cos\alpha \end{bmatrix}$

SECTION D

Q32. (i) If y $= \frac{1}{1+x^{a-b}+x^{c-b}} + \frac{1}{1+x^{b-c}+x^{a-c}} + \frac{1}{1+x^{b-a}+x^{c-a}}$, then find $\frac{dy}{dx}$.

(ii) if $y = a(\theta + sin\theta),\ \ x = a(1 - cos\theta)\ then\ find\ \frac{d^2y}{dx^2}$

Q33. Evaluate $\int \frac{(\sin 2x - \cos x)dx}{\sqrt{10-\cos^2 x-4\sin x}}$

Q34. Determine graphically the minimum value of the objective function $\boldsymbol{Z = -50x + 20y}$

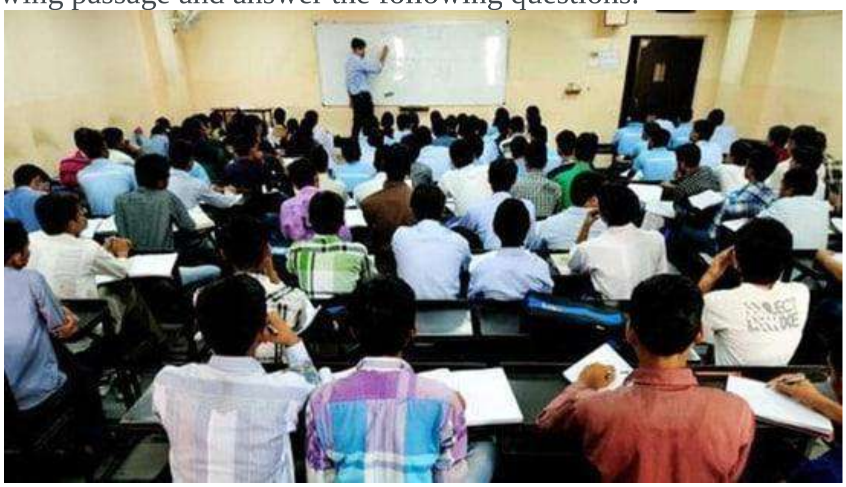
Subject to the constraints $\boldsymbol{2x - y \geq -5,\ \ 3x + y \geq 3\ ,\ \ 2x - 3y \leq 12,\ \ \ x, y \geq 0}$

Q35. If A= $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$ then find A^{-1} and using A^{-1} solve the following system of linear equations.

$\boldsymbol{x + y + 2z = 0,\ x + 2y - z = 9\ , x - 3y + 3z = -14}$

SECTION E
CASE STUDY BASED QUESTIONS

Q36. Read the following passage and answer the following questions.



In coaching institutes, the students not only get academic guidance but also they get to know about career options and right goals as per their interest and academic record.

A coaching institute conduct classes in two sections A and B and fees for rich and poor children are different. In section A. there are 20 poor and 5 rich children and total monthly collection is 19,000, where as in section B, there are 5 poor and 25 rich children and total monthly collection is 26,000.

(i) (a) If x and y be the fees for rich and poor children respectively, then express the information provided in problem in system of linear equation.

(b) Express the system of linear equations obtained in (a) as matrix equation.

(ii) Find the fees for rich and poor children.

Q37. A tank, as shown in the figure below, formed using a combination of a cylinder and a cone, offers better drainage as compared to a flat bottomed tank. If slant height of the conical part is given as L, then answer the following questions.

(i) Find the value of radius of conical part in terms of height of the cone.
(ii) Find the maximum volume of the cone.
(iii) Show that semi vertical angle of the cone is $\tan^{-1}\sqrt{2}$

Q38. In a school project Virat was asked to construct a Triangle ABC in which two angles B and C are given by $\tan^{-1}(\frac{1}{2})$ and $\tan^{-1}(\frac{1}{3})$ respectively.

(i) Find the value of sinB

(ii) Find the value of cos C.

(iii) Find the value of B+C