



VENKATESHWAR INTERNATIONAL SCHOOL
Sector – 10, Dwarka, New Delhi –110075
HALF YEARLY EXAMINATION (2024 – 2025)
CLASS – XII
MATHEMATICS

Time: 3 hrs

Max. Marks: 80

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XII LD (12)

General Instructions:

1. All questions are compulsory.
2. The paper consists of 4 Sections A, B, C and D. Section A contains 18 questions (1-18) mcqs of 1 mark each, question 19 and 20 assertion and reasoning 1 mark each. Section B contains 5 questions of 2 marks each, Section C contains 6 questions of 3 marks each and Section D contains 4 long questions of 5 marks each. Section E contains 3 case study based questions of 4 marks each.
3. There is no overall choice but internal choices have been provided in some questions. You have to attempt only one of the choices in such questions.
4. Use of calculator is not permitted. However you may ask for logarithmic tables if required.

Section - A
(OBJECTIVE)

Multiple Choice Questions (1 mark each)

- Q 1. The value of $2\cos^{-1}\left(\frac{-1}{2}\right) + 2\sin^{-1}\left(\frac{-1}{2}\right) - \cos^{-1}(-1)$ is
- a.) 0 b.) $\frac{\pi}{2}$ c.) π d.) 2π
- Q 2. $\int \frac{e^{x+3}}{(x+4)^2} dx$ is equal to
- a.) $\frac{e^x}{x+4} + C$ b.) $\frac{e^{-x}}{x+4} + C$ c.) $\frac{e^x}{x-4} + C$ d.) $\frac{e^{2x}}{x+4} + C$
- Q 3. If the function $f(x) = 2x^2 - kx + 5$ is increasing on $[1, 2]$, then k lies in the interval
- a.) $(-\infty, 8)$ b.) $(4, \infty)$ c.) $(-\infty, 4)$ d.) $(8, \infty)$
- Q 4. The domain of the function f such that $f(x) = \cos^{-1}(2x - 1)$ is
- a.) $[0, 1]$ b.) $[-1, 1]$ c.) $[0, \pi]$ d.) $(-1, 1)$
- Q 5. The principal value of $\sin^{-1}\left(\cos\left(\frac{\pi}{9}\right)\right)$ is
- a.) $\frac{\pi}{9}$ b.) $\frac{5\pi}{9}$ c.) $-\frac{5\pi}{9}$ d.) $\frac{7\pi}{18}$
- Q 6. The function $f(x) = \frac{4-x^2}{4x-x^3}$ is
- a.) discontinuous at only one point b.) discontinuous at exactly two points
- c.) discontinuous at exactly three points d.) None

- Q 7. If A is a square matrix such that $(A - 2I)(A + I) = 0$, then A^{-1} is equal to
 a.) $\frac{A-I}{2}$ b.) $\frac{A+I}{2}$ c.) $2(A - I)$ d.) $2A + I$
- Q 8. The number of all possible matrices of order 2x2 with each entry 3, 2 or 1 is
 a.) 18 b.) 27 c.) 81 d.) 512
- Q 9. If $A = \begin{bmatrix} 0 & a & 1 \\ -1 & b & 1 \\ -1 & c & 0 \end{bmatrix}$ is a skew symmetric matrix, then the value of $(a + b + c)^2$ is
 a.) 1 b.) 0 c.) 4 d.) 10
- Q10. If A is 3 x 3 matrix, then the value of k if $\text{Det}(A^{-1}) = (\text{Det} A)^k$ is
 a.) -1 b.) 3 c.) 9 d.) not defined
- Q 11. If $A = \begin{bmatrix} 2 & \beta & -3 \\ 0 & 2 & 5 \\ 1 & 1 & 3 \end{bmatrix}$, then A^{-1} exists, if
 a.) $\beta = 2$ b.) $\beta \neq 2$ c.) $\beta \neq -2$ d.) $\beta \neq -\frac{8}{5}$
- Q 12. If $A = \begin{bmatrix} 1 & 2 & -1 \\ -1 & 1 & 2 \\ 2 & -1 & 1 \end{bmatrix}$ then $|\text{Adj}(\text{adj}A)|$ is
 a.) 14^4 b.) 14^3 c.) 14^2 d.) 14
- Q 13. The number of corner points of feasible region determined by the constraints $x - y \geq 0, 2y \leq x + 2, x \geq 0, y \geq 0$ is
 a.) 2 b.) 3 c.) 4 d.) 5
- Q 14. The objective function $Z = ax + by$ of an LPP has maximum value 42 at (4,6) and minimum value 19 at (3,2). The value of a and b is
 a.) $a = 9, b = 1$ b.) $a = 5, b = 2$ c.) $a = 3, b = 5$
 d.) $a = 5, b = 3$
- Q 15. Principal branch of $\csc^{-1} x$ is equal to
 a.) $(-\frac{\pi}{2}, \frac{\pi}{2})$ b.) $[-\frac{\pi}{2}, \frac{\pi}{2}]$ c.) $[0, \pi] - \{\frac{\pi}{2}\}$ d.) $[-\frac{\pi}{2}, \frac{\pi}{2}] - \{0\}$
- Q 16. The derivative of $\cos^{-1}(2x^2 - 1)$ w.r.t. $\cos^{-1} x$ is
 a.) 2 b.) $1 - x^2$ c.) $\frac{2}{x}$ d.) $\frac{-1}{2\sqrt{1-x^2}}$
- Q 17. If x is real, the minimum value of $x^2 - 8x + 17$ is
 a.) -1 b.) 0 c.) 1 d.) 2
- Q 18. If $\int_0^{40} \frac{dx}{2x+1} = \log K$, then value of K is
 a.) 3 b.) $9/2$ c.) 9 d.) 20

ASSERTION AND REASONING BASED QUESTIONS (1 + 1 mark)

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option out of the following choices

- a.) Both A and R are true and R is correct explanation of A.
- b.) Both A and R are true but R is not the correct explanation of A.
- c.) A is true but R is false.
- d.) A is false and R is also false.

Q 19.

Statement 1: $\text{adj}(\text{adj } A) = A$

Statement 2: $|\text{adj } A| = |A|$

Q 20. Statement 1: If $f'(x) = 10x + 7$ and $f(1) = 20$, then $f(x) = 5x^2 + 7x + 8$

Statement 2: $\int f'(x) dx = f(x) + C$

SECTION - B (2 marks each)

Q 21. If $A = \frac{1}{\pi} \begin{bmatrix} \sin^{-1}(\pi x) & \tan^{-1}\left(\frac{x}{\pi}\right) \\ \sin^{-1}\left(\frac{x}{\pi}\right) & \cot^{-1}(\pi x) \end{bmatrix}$ and $B = \frac{1}{\pi} \begin{bmatrix} -\cos^{-1}(\pi x) & \tan^{-1}\left(\frac{x}{\pi}\right) \\ \sin^{-1}\left(\frac{x}{\pi}\right) & -\tan^{-1}(\pi x) \end{bmatrix}$, then find $A - B$.

OR

If $A = \begin{bmatrix} 1 & -1 \\ 3 & 6 \end{bmatrix}$, find $\text{adj}(\text{adj } A)$.

Q 22. Evaluate: $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \pi x^3 e^{\sin^2 x} dx$

OR

Evaluate: $\int_{-1}^1 e^{|x|} dx$

Q 23. Evaluate: $\int \frac{2^{x+1} - 5^{x-1}}{10^x} dx$

Q 24. The amount of pollution content added in air in a city due to x -diesel vehicles is given by $P(x) = 0.005x^3 + 0.02x^2 + 30x$. Find the marginal increase in pollution content when 3 diesel vehicle are added.

Q 25. Solve for x : $\cos(\tan^{-1} x) = \sin(\cot^{-1} \frac{3}{4})$.

OR

Prove that: $\cos\left(\sin^{-1} \frac{3}{5} + \cot^{-1} \frac{3}{2}\right) = \frac{6}{5\sqrt{13}}$

SECTION C (3 marks each)

Q 26. If $y = e^{a \cos^{-1} x}$, $-1 < x < 1$, then show that $(1-x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} - a^2y = 0$.

Q 27. Find the minimum value of $(ax + by)$, where $xy = c^2$.

Q 28. Find graphically, the maximum value of $Z = 2x + 5y$, subject to constraints given below:

$$2x + 4y \leq 8, 3x + y \leq 6, x + y \leq 4, x \geq 0, y \geq 0.$$

Q 29. Prove that $|A| = \begin{vmatrix} x & \sin A & \cos A \\ -\sin A & -x & 1 \\ \cos A & 1 & x \end{vmatrix}$ is independent of A. If $|A| = 8$, then find the value of x.

OR

If $[1 \ x \ 1] \begin{bmatrix} 1 & 3 & 2 \\ 0 & 5 & 1 \\ 0 & 3 & 2 \end{bmatrix} \begin{bmatrix} x \\ 1 \\ -2 \end{bmatrix} = 0$, then what is the value of x?

Q 30. Evaluate: $\int_0^{\pi/2} \sin 2x \tan^{-1}(\sin x) dx$

OR

Evaluate: $\int \sec^3 x dx$

Q 31. Differentiate w.r.t x: $\sin^{-1} \left(\frac{2^{x+1} \cdot 3^x}{1+(36)^x} \right)$

SECTION - D (5 marks each)

Q 32. Evaluate: $\int_{-1}^2 |x^3 - x| dx$

OR

Evaluate: $\int_0^{3/2} |x \cos \pi x| dx$

Q 33. Evaluate: $\int \frac{x^3+x}{x^4-9} dx$

OR

Evaluate: $\int \frac{x+2}{\sqrt{x^2+5x+6}} dx$

Q 34. Find the derivative of the following function $f(x)$ w. r. t. x at $x = 1$:

$$f(x) = \cos^{-1} \left(\sin \sqrt{\frac{1+x}{2}} \right) + x^x$$

Q 35.(i) Find the area bounded by the curve $y=x|x|$ and the ordinates $x=-1$ and $x = 1$.

- (ii) Sketch the region $\{(x, 0): y = \sqrt{4 - x^2}\}$ and x-axis. Find the area of the region using integration.

CASE STUDY – 1(1+1+2 marks)

- Q 36. A cable network provider in a small town has 500 subscribers and he used to collect Rs 300 per month from each subscriber. He proposes to increase the monthly charges and it is believed from past experience that for every increase of Rupee 1, one subscriber will discontinue the service.

Based on given information, answer the following questions:

- i.) If Rupee x is the monthly increase in subscription amount, then find the number of remaining subscribers.
- ii.) Find the total revenue 'R' (in Rs).
- iii.) Find the number of subscribers which gives the maximum revenue.

OR

What is the maximum revenue generated?

CASE STUDY – 2 (1+1+2 marks)

- Q 37. A diet is to contain 30 units of Vitamin A, 40 units of Vitamin B and 20 units of Vitamin C. Three types of foods F_1, F_2 and F_3 are available. One unit of Food F_1 contains 3 units of Vitamin A, 2 units of Vitamin B and 1 unit of Vitamin C. One unit of Food F_2 contains 1 units of Vitamin A, 2 units of Vitamin B and 1 unit of Vitamin C. One unit of food F_3 contains 5 units of Vitamin A, 3 units of Vitamin B and 2 units of Vitamin C. Based on the above information answer the following questions:

- (i) If the diet contains x units of Food F_1 , y units of Food F_2 and z units of Food F_3 . What is the matrix equation representing the above situation.
- (ii) If A is the coefficient matrix in above situation, then what is the value of $|\text{adj } A|$?
- (iii) What are the values of x, y, z in given situation?

OR

What is the value of $|A^{-1}|$.

CASE STUDY – 3 (2 + 2 marks)

- Q 38. The equation of the path traced by a roller-coasters is given by the polynomial

$$F(x) = a(x+a)(x+1)(x-3). \text{ If the roller - coaster crosses } y - \text{ axis at a point}$$

$(0, -1)$, Answer the following questions:

- (i) Find the value of a .
- (ii) Find $F''(x)$ at $x = 1$.