ITL PUBLIC SCHOOL
MID TERM EXAMINATION(2025-26)

Date:.19.09.25 **Class: XII**

MATHEMATICS (041) (SET B)

Time: 3hrs **M. M: 80**

General Instructions:
- This question paper contains five sections – A, B,C, D and E. Each part is compulsory.
- Section - A has 20 MCQ of 1 mark each.
- Section – B has 5 short answer type questions of 2 marks each.
- Section – C has 6 questions of 3 marks each.
- Section - D has 4 long answer type questions of 5 marks each.
- Section -.E has 3 Case Study questions of 4 marks each.

SECTION - A

1 If the points (2, - 3), (λ, -1) and (0, 4) are collinear, then the value of λ is 1

(a) $\frac{10}{7}$ (b) $\frac{7}{10}$ (c) $\frac{3}{10}$ (d) 1

2 If $y = \tan^{-1}\left(\frac{\sin x + \cos x}{\cos x - \sin x}\right)$, then $\frac{dy}{dx}$ is equal to 1

(a) $\frac{1}{2}$ (b) 0 (c) 1 (d) -1/2

3 The matrix $A = \begin{bmatrix} 2 & 1 & 3 \\ 4 & -1 & 0 \\ -7 & 2 & 1 \end{bmatrix}$ is 1

(a) Singular matrix (b) Non-singular

(c) Symmetric matrix (d) Skew-symmetric matrix

4 The function $f(x) = e^{-|x|}$ is 1

(a) continuous everywhere but not differentiable at x = 0

(b) continuous and differentiable everywhere

(c) not continuous at x = 0

(d) Neither continuous nor differentiable anywhere

5 If the value of a third order determinant is 12, then the value of the determinant formed by replacing 1

each element by its cofactor will be

(a) 1728 (b)0 (c) 12 (d)144

6 $\int e^x(1 - \cot x + \cot^2 x)dx =$ 1

(a) $e^x \cot x + C$ (b) $-e^x \cot x + C$ (c) $e^x \operatorname{cosec} x + C$ (d) $-e^x \operatorname{cosec} x + C$

7 The value of $\tan^{-1}\left(\tan\frac{5\pi}{6}\right) + \cos^{-1}\left(\cos\frac{13\pi}{6}\right)$ is 1

(a) 0 (b) $\frac{2\pi}{3}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{3}$

8 Find the intervals in which the function $f(x) = x^2 - 4x + 6$ is strictly increasing 1

(a) $(2, \infty)$ (b) $(-\infty, 2)$ (c) $(-\infty, 2) \cup (2, \infty)$ (d) $(-\infty, 2] \cup (2, \infty)$

9 The corner points of the feasible region determined by the system of linear constraints are (0, 10), (5, 5), (15, 15), (0, 20). Let Z = px + qy, where p, q > 0. Condition on p and q so that the maximum of Z occurs at both the points (0,10) and (5, 5) is 1

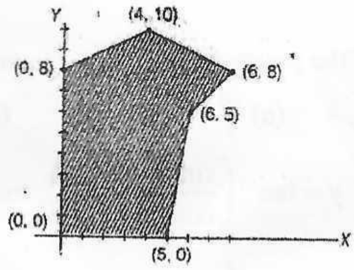
(a) p = q (b) p = 2q (c) q = 2p (d) q = 3p

10 A particle moves along the curve $x^2 = 2y$. The point at which, ordinate increases at the same rate as the abscissa is ________ 1

(a) (1, 2) (b) $\left(\frac{1}{2}, 1\right)$ (c) $\left(\frac{1}{2}, \frac{1}{2}\right)$ (d) $\left(1, \frac{1}{2}\right)$

11 The feasible solution for a LPP is shown in following figure. 1

Let Z = 3x - 4y be the objective function. Minimum of Z occurs at



(a)(0,0) (b)(0, 8) (c) (5,0) (d) (4,10)

12 The least and maximum value of $f(x) = x^3 - 6x^2 + 9x$ in [0,6] are 1

(a) 3,4 (b) 0,6 (c) 0,54 (d) 3,6

13 $\int_{-5}^{5} |x-2|\, dx$ is equal to 1

(a) -4 (b) 18 (c) 29 (d) 0

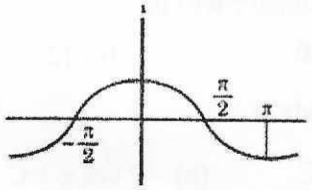
14 $\int_{0}^{\pi/2} \frac{\sin^{3/2} x}{\sin^{3/2} x + \cos^{3/2} x} dx$ equals to 1

(a) π (b) $\pi/2$ (c) $\pi/3$ (d) $\pi/4$

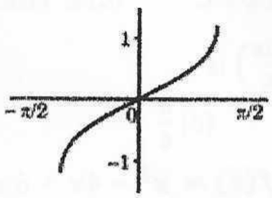
15 Which of the following functions is decreasing on $\left(0, \frac{\pi}{2}\right)$. 1

(a) sin 2x (b) tan x (c) cos x (d) cos 3x

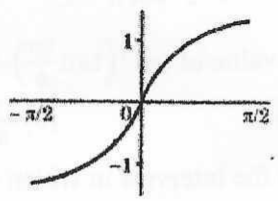
16 The graph of trigonometric function is as shown. Which of the following represent the graph of its inverse. 1



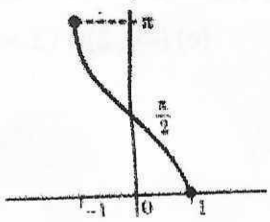
(A)



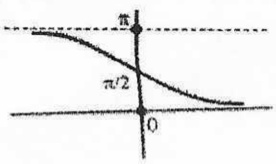
(B)



(C)



(D)



17 Area bounded by the curve $y = x^3$, the x-axis and the ordinates x = - 1 and x = 1 is 1

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{15}{4}$ (d) $\frac{17}{4}$

18 If p and q are respectively the order and degree of the differential equation $\frac{d}{dx}\left(\frac{dy}{dx}\right)^3 = 0$ then (p - q) is 1

(a) 1 (b) -1 (c) 2 (d) 3

ASSERTION-REASON BASED QUESTIONS(19,20)

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

19 Assertion (A): The number of arbitrary constants in the general solution of a differential equation of order three is 3. 1

Reason (R): The number of arbitrary constants in the general solution of a differential equation is equal to the degree of the equation.

20 Assertion (A): If A is a symmetric matrix, then B'AB is also symmetric. 1

Reason (R): (ABC)' = C'B'A'

SECTION - B

21 Evaluate: $\int \frac{\sin x}{\sin(x-a)}\,dx$ 2

OR

Evaluate: $\int x \cot^{-1} x\, dx$

22 Show that f(x) = | x - 5 | is continuous at x = 5. 2

OR

Determine if $f(x) = \begin{cases} x^2 \sin\frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ is a continuous function?

23 Solve the following Differential equation: $\frac{dy}{dx} + y\cot x = 4x\,\mathrm{cosec}\,x$ 2

24 Find the value of x such that $\begin{bmatrix} 1 & x & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 15 & 3 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix} = 0$ 2

25 Prove that $f(\theta) = \frac{4\sin\theta}{2+\cos\theta} - \theta$ is an increasing function of θ in $\left[0, \frac{\pi}{2}\right]$. 2

SECTION - C

26 If $y = e^{a \sin^{-1}x}$, $-1 \le x \le 1$, then show that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2y = 0$. 3

OR

If $y = (\tan^{-1}x)^2$, then show that $(x^2+1)^2\frac{d^2y}{dx^2} + 2(x^3+x)\frac{dy}{dx} = 2$.

27 Evaluate: $\int \frac{2x+3}{\sqrt{x^2+4x+1}}dx$ 3

28 If $\sqrt{1-x^4} + \sqrt{1-y^4} = a(x^2-y^2)$, Prove that $\frac{dy}{dx} = \frac{x}{y}\sqrt{\frac{1-y^4}{1-x^4}}$. 3

29 Solve the following Differential Equation: $\frac{dy}{dx} - 2xy = 3x^2e^{x^2}$; $y(0) = 5$ 3

OR

Solve the following Differential Equation: $x^2dy + y(x+y)dx = 0$

30 Prove that $\tan\left\{\frac{\pi}{4}+\frac{1}{2}\cos^{-1}\frac{a}{b}\right\}+\tan\left\{\frac{\pi}{4}-\frac{1}{2}\cos^{-1}\frac{a}{b}\right\}=\frac{2b}{a}$ 3

31 Express the following matrix as the sum of a symmetric and skew-symmetric matrix and verify your result: $A = \begin{bmatrix} 3 & 2 & 7 \\ 1 & 4 & 3 \\ -2 & 5 & 8 \end{bmatrix}$. 3

SECTION -D

32 Using integration ,find the area bounded by the curve: $9x^2 + 25y^2 = 225$ 5

33 Given $A=\begin{bmatrix} 5 & 0 & 4 \\ 2 & 3 & 2 \\ 1 & 2 & 1 \end{bmatrix}$ $B^{-1}=\begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$. Compute $(AB)^{-1}$. 5

OR

Determine the product $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ and use it to solve the system of equations:

$$x - y + z = 4,\quad x - 2y - 2z = 9,\quad 2x + y + 3z = 1$$

34 Solve the following LPP graphically: 5

Minimise and Maximise $Z = 5x + 10y$ subject to

$x + 2y \le 120$, $x + y \ge 60$, $x - 2y \ge 0$, $x, y \ge 0$

35 Prove that : $\int_0^{\pi} \frac{x}{1-\cos\alpha \sin x}dx = \frac{\pi(\pi-\alpha)}{\sin\alpha}$ 5

OR

Evaluate: $\int_0^a \sin^{-1}\sqrt{\frac{x}{a+x}}dx$

SECTION -E

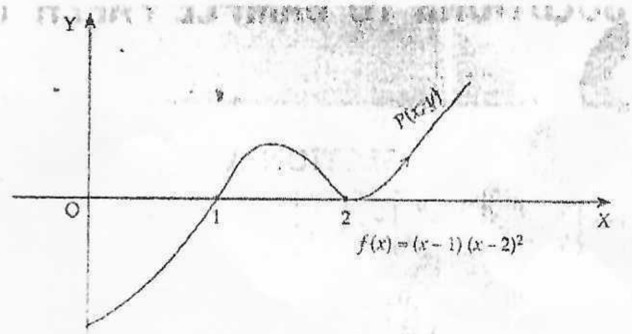
36 A particle is moving along the curve represented by the polynomial $f(x) = (x-1)(x-2)^2$, as shown in the figure given below: 4

Based on the above information, answer the following questions:

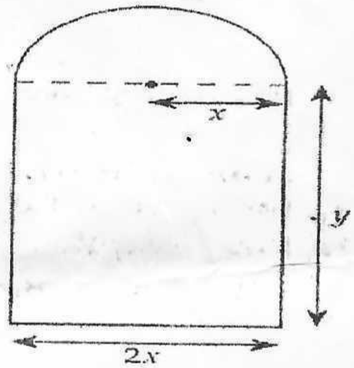
(i) Find the critical points of function f(x).

(ii) Find the interval where f(x) is strictly increasing.

(iii) Find the local minimum value of f(x).



37 Mr. Dhar is an architect. He designed a building and provided an entry door in the shape of a rectangle surmounted by a semicircular opening. The perimeter of the door is 10m. 4



(i) If 2x metres and y metres be the breadth and the height of the rectangular part of the door, then find a relation between x and y.

(ii) Find the area enclosed by the door in terms of x.

(iii) To allow maximum airflow inside the building, what should be the width of the door?

38 The management committee of a residential colony decided to award some of its members (say x) for honesty, some (say y) for helping others means cooperation and some others (say z) for supervising the workers to kept the colony neat and clean. The sum of all the awardees is 12. Three times the sum of awardees for cooperation and supervision added to two times the number of awardees for honesty is 33. The sum of the number of awardees for honesty and supervision is twice the number of awardees for helping. Using matrix method find x ,y and z. 4